A Turbulent Transport Model for Free-Shear Flows

S. Biringen* and K. Abdol-Hamid†
University of New Hampshire
Durham, New Hampshire

Introduction

THE central problem in turbulence modeling is the empirical formulation of higher order terms that appear in turbulent transport equations used for closure; one such term represents turbulent diffusion. Vollmers and Rotta,[1] Hanjalic and Launder,[2] and Launder and Morse,[3] among others, have used the gradient transport hypothesis to model this term. It is interesting to note that such models have not been successful in calculating the characteristics of free-shear flows. In particular, the calculations of Launder and Morse[3] using a stress-equation turbulence model predict a higher rate of spread for the axisymmetric free jet than a jet with swirl. In addition, this model predicts a faster rate of spread for the round free jet than the two-dimensional jet. Although Launder and his coworkers attribute this failure to the modeling of the rate-of-dissipation equation, recent work by Riberio and Whitelaw[4] lends evidence to the gradient transport form of the modeled kinetic-energy diffusion as a possible source of these ambiguities. This seems to be the more likely explanation if one considers that the calculations of Vollmers and Rotta,[1] employing a two-equation model of turbulence with gradient transport, yield a value of the turbulent kinetic energy on the centerline of a round jet about 30% lower than the experimental value. Note that in their calculations, Vollmers and Rotta[1] used a length-scale equation rather than a dissipation rate equation, as in the case of Launder and Morse.[3]

The experiments of Riberio and Whitelaw[4] indicate that the mechanism of turbulent diffusion includes nonlocal effects that cannot be represented by gradient diffusion. A nonlocal model based on the bulk-convection hypothesis of Townsend[5] was advanced by Bradshaw et al.[6] for boundary-layer flows and used by Biringen[7] to calculate free-shear flows. However, the use of the bulk-convection hypothesis renders the governing equations to be of hyperbolic type and, in some cases, the numerical stability of such equations may degenerate where the local eigenvalues become very stiff. This is particularly reflected in the kinetic-energy equation when such a model is used to calculate the round jet.[7]

The major aim of the present work is to formulate a more accurate and physically plausible diffusion model that includes the effects of large-eddy motion on turbulent transport. We propose to achieve this by incorporating bulk convection and gradient diffusion; a simplified form of this model was used by Biringen and Levy[8] for the calculation of boundary layers. The two-equation model of turbulence thus obtained is used in self-similar form to calculate axisymmetric and plane two-dimensional jets. Comparisons with experiments as well as with the calculations of Vollmers and Rotta[1] provide a means of evaluating the proposed model for turbulent transport in free-shear flows.

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*Associate Professor, Department of Mechanical Engineering, Member AIAA.
†Graduate Student, Department of Mechanical Engineering, Member AIAA.

Turbulence Model and Closure Assumptions

The governing equations are written with respect to the Cartesian coordinates x, y, and z. The mean velocity components are U, V, and W, and the fluctuating velocity components are u, v, and w, respectively; an overbar denotes a time average. The flow is assumed to be steady, two-dimensional, incompressible, and have constant properties. The governing equations are simplified by the thin-shear flow approximations. The shear stress is calculated via the Kolmogorov-Prandtl formula:

$$\overline{uv}=A_1 k^{1/2} L\frac{\partial U}{\partial y} \tag{1}$$

Here, L is the turbulent length scale, k the turbulent kinetic energy, and $A_l \simeq -0.5$. Transport equations are used for k and L (see Ref. 9 for a detailed discussion), and the related closure assumptions are made following Refs. 7 and 8. In particular, the diffusion terms in the governing equations are modeled by a combination of bulk convection and gradient diffusion and written as

$$\overline{kv}=V_k k+C_6 k^{1/2}L\frac{\partial k}{\partial y},\quad \overline{Lv}=V_L L+C_6 k^{1/2}L\frac{\partial L}{\partial y} \tag{2}$$

where C_6 is the diffusion constant given as $C_6=-0.8$, and V_k and V_L are the bulk velocities for k and L, respectively. These are prescribed as [6,7]

$$V_k=|\overline{uv}|^{1/2}_{\max}f_1(y),\quad V_L=|\overline{uv}|^{1/2}_{\max}f_2(y) \tag{3}$$

where $f_1(y)$ and $f_2(y)$ are functions to be determined empirically. Following Biringen,[7] we assume $f_1(y)=f_2(y)$, so that once $f_1(y)$ is prescribed the turbulence model equations will constitute a closed system. The resulting set of partial differential equations can be transformed into ordinary differential equations if the flow is treated as a self-similar flow for which the necessary conditions are well established.[5,9] I, f, g, g_{12}, and h are denoted as the nondimensional form of V, U, $\sqrt{k}$, $\overline{uv}$, and L; these are defined as a function of the independent variable, $\eta=y/x$. The ordinary differential equations thus obtained are

continuity:

$$\frac{dI}{d\eta}+\frac{Ir}{\eta}-\left(\eta\frac{df}{d\eta}+\frac{r+1}{2}f\right)=0 \tag{4}$$

streamwise momentum:

$$(I-f\eta)\frac{df}{d\eta}+\frac{dg_{12}}{d\eta}+\frac{rg_{12}}{\eta}-\frac{r+1}{2}f^2=0 \tag{5}$$

transport:

$$\alpha_1\frac{d^2\Gamma}{d\eta^2}+[\alpha_2+\alpha_3+\alpha_4]\frac{d\Gamma}{d\eta}+\alpha_5\frac{r\Gamma}{\eta}+(r+1)\alpha_6 f\Gamma+\alpha_7=0 \tag{6}$$

where Γ can be either g or h; here $r=1$ for axisymmetric flow and $r=0$ for two-dimensional flow. Table 1 summarizes the expressions for α_1-α_7 corresponding to these transport equations.

Finally, the function f_1 is shown in Fig. 1; note that the solutions are essentially insensitive to the prescribed slope of f_1 for $y/x>0.075$.

Results and Discussion

Equations (4-6), along with the Kolmogorov-Prandtl formula, were integrated by the fourth-order Runge-Kutta method for the cases of axisymmetric and plane, two-

Table 1 Coefficients for the transport equation [Eq. (8)]

Coefficient equation	α_1	α_2	α_3	α_4	α_5	α_6	α_7
Kinetic energy ($\Gamma=g$)	$-2C_6g^2h$	$(I-f\eta)2g$	$-2C_6\dfrac{dg^2h}{d\eta}$	$2f_1Dg$	$-2C_6gh\dfrac{dg}{d\eta}+f_1g^2D$	g	$g_{12}\dfrac{df}{d\eta}+C_1\dfrac{g}{h}+\dfrac{df_1}{d\eta}g^2D$
Length scale ($\Gamma=h$)	$-C_6gh$	$(I-f\eta)$	$-C_6\left(5h\dfrac{dg}{d\eta}+g\dfrac{dh}{d\eta}\right)$	f_1D	$-g\dfrac{dh}{d\eta}$	$\frac{1}{2}(1-r)$	$-h\dfrac{df}{d\eta}+bg$

N.B: $D=|g_{12}|^{1/2}_{max}$, $C_1=0.165$, $b=0.033$.

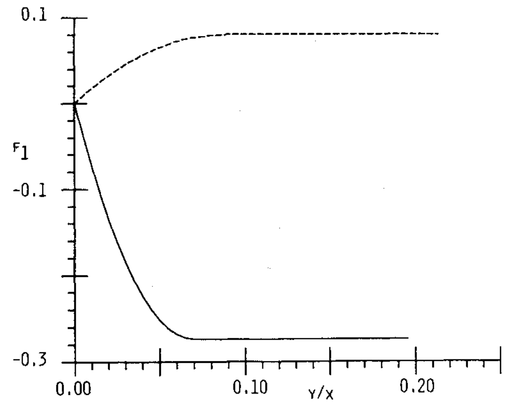

Fig. 1 Kinetic-energy bulk-convection function: ——, axisymmetric; ---, plane jet.

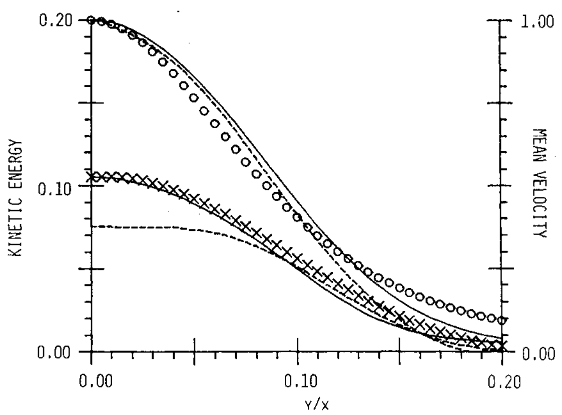

Fig. 2 Axisymmetric jet $\tilde{U}$ and k profiles; o, measured $\tilde{U}$ profile from Ref. 10; ×, measured k profile from Ref. 10;——, present calculation; ---, Vollmers-Rotta[1] results.

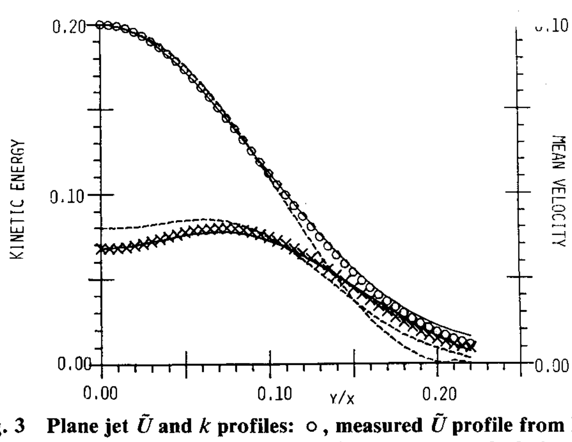

Fig. 3 Plane jet $\tilde{U}$ and k profiles: o, measured $\tilde{U}$ profile from Ref. 11; ×, measured k profile from Ref. 11;——, present calculation; ---, Vollmers-Rotta[1] results.

dimensional free jets. Initial conditions were obtained from the measurements of Wygnanski and Fiedler[10] and Bradbury,[11] respectively. Symmetry requirements on the jet centerline were also imposed. The turbulent model constants C_1, C_6, b, and A_1 were kept at their respective values used in previous work[1,2,7] throughout the calculations.

A comparison of the predicted $\tilde{U}$ and k profiles for the axisymmetric jet with the calculations of Ref. 1 and experimental results of Ref. 10 is shown in Fig. 2. Here a tilde denotes a variable nondimensionalized by the jet centerline velocity. Although both calculations seem to depart from the experimental $\tilde{U}$ profile for $y/x>0.10$, the discrepancy displayed by the Vollmers-Rotta results is very substantial, especially toward the edges of the jet. The overall agreement of the present results with the measured $\tilde{U}$ profile is good. Also presented in Fig. 2 are the variations of $\tilde{k}$ across the flow displaying excellent agreement between the results presented herein and the experiment. The calculations of Vollmers and Rotta, however, underpredict this quantity by about 30% at the jet centerline. Note that with the use of any value of $\tilde{k}>0.075$ on the jet centerline as an initial condition, the gradient transport model yields solutions completely incompatible with the observed and expected variations of $\tilde{k}$ for $y/x>0.10$. As evidenced from Fig. 2, with the inclusion of the bulk convection into turbulent diffusion, the present two-equation model accurately predicts the variations of the turbulent kinetic energy across the flow. Thus, it can be asserted that the apparent inability of gradient-diffusion models to predict the axisymmetric jet can be overcome by modeling turbulent diffusion in terms of both gradient transport and bulk convection. A likely explanation for this observation is that, while the gradient diffusion represents the "up gradient" part of the process, the effect of the large scales of motion is reflected only by bulk convection.

The variations of $\tilde{U}$ and $\tilde{k}$ for the plane jet are presented in Fig. 3. Here, the present results are compared with the experiments of Bradbury[12] and with the calculations of Vollmers and Rotta; excellent agreement between the calculations presented herein and the experiment is observed. The significant deviation of the Vollmers-Rotta results from the measured profiles for both quantities is also apparent in this figure.

In summary, the model presented herein was able to predict the variations of $\tilde{U}$ and $\tilde{k}$ across axisymmetric and plane free jets with considerable accuracy. These calculations in particular, demonstrated that a two-equation model of turbulence, with the diffusion terms modeled by a combination of bulk convection and gradient transport, leads to significant improvements over the gradient-diffusion model of Ref. 1.

Acknowledgments

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References

[1]Vollmers, H. and Rotta, J. C., "Similar Solutions of Mean Velocity, Turbulent Energy and Length Scale Equation," *AIAA Journal*, Vol. 15, May 1977, pp. 714-720.

[2]Hanjalic, K. and Launder, B. E., "A Reynolds Stress Model of Turbulence and Its Application to Thin Shear Flow," *Journal of Fluid Mechanics*, Vol. 52, 1972, pp. 609-638.

[3]Launder, B. E. and Morse, A., "Numerical Prediction of Axisymmetric Free Shear Flows with a Reynolds Stress/Closure," *Turbulent Shear Flows I*, edited by F. Durst et al., Springer-Verlag, Berlin, 1979, pp. 279-294.

[4]Riberio, M. M. and Whitelaw, J. H., "Coaxial Jets with and without Swirl," *Journal of Fluid Mechanics*, Vol. 95, 1980, pp. 769-795.

[5]Townsend, A. A., *The Structure of Turbulent Shear Flow*, Cambridge University Press, Cambridge, England, 1956.

[6]Bradshaw, P., Ferriss, D. H., and Atwell, N. P., "Calculations of Boundary-Layer Development Using the Turbulent Energy Equation," *Journal of Fluid Mechanics*, Vol. 28, 1967, pp. 593-616.

[7]Biringen, S., "Calculation of Axisymmetric Jets and Wakes with a Three-Equation Model of Turbulence," *Journal of Fluid Mechanics*, Vol. 86, 1978, pp. 745-759.

[8]Biringen, S. and Levy J., "Calculation of Two Dimensional Turbulent Boundary Layers," *AIAA Journal*, Vol. 16, Sept. 1978, pp. 1016-1019.

[9]Adbol-Hamid, K. S. and Biringen, S., "Self-Similar Solution of Turbulent Jets Using Second-Order Closure Models," Mechanical Engineering Dept., Univ. of New Hampshire, Durham, Tech. Rept. ME-TFA-84011, 1984.

[10]Wygnanski, I. and Fiedler, H., "Some Measurements in the Self-Preserving Jet," *Journal of Fluid Mechanics*, Vol. 38, 1969, pp. 874-912.

[11]Bradbury, L. J. S., "The Structure of a Self-Preserving Turbulent Plane Jet," *Journal of Fluid Mechanics*, Vol. 23, 1965, pp. 31-64.

Reduction of Background Noise Induced by Wind Tunnel Jet Exit Vanes

Ruth M. Martin,* Thomas F. Brooks† and Danny R. Hoad‡

NASA Langley Research Center, Hampton, Virginia

THE NASA Langley 4×7 m wind tunnel, when operating in the open-throat mode, exhibits a phenomenon typical of many open-jet tunnels—development of low-frequency flow pulsations at certain velocity ranges. This pulsation, sometimes called "wind tunnel pumping," is thought to be created by the interaction of the unstable shear layer formed at the jet exit and acoustic waves radiated from the region of impingement of the shear layer on the flow collector. Vortices formed in the shear layer at the jet exit travel downstream and strike the collector, causing a pressure fluctuation that then travels back upstream, interacting with the jet shear layer and causing additional vortex shedding at the jet exit. When this shedding occurs at critical frequencies related to path lengths in the tunnel circuit, resonances can occur that enhance the phenomenon. Such pulsations are of serious concern because they affect the aerodynamic quality of the flow and thus the quality of the resulting data. In the NASA Langley 4×7 m tunnel, the open-throat pulsation problem occurs at three distinct ranges of tunnel dynamic pressure as shown in Fig. 1. In order to conduct a test in a smooth flow environment, the test speed must be within the limited ranges where the pulsation is at a minimum. For most tests, this is impractical due to the operational requirements and aims of the test program.

Passive devices to inhibit the development of such pulsation problems have been applied in various facilities in the past. The devices generally attempt to introduce random turbulence, thereby delaminarizing and destroying the shear layer vortex structure. Vanes, tabs, or teeth that protrude into the airstream around the jet exit have been shown to effectively reduce the magnitude of these flow pulsations.[1-4]

Six configurations of jet exit passive devices were tested and evaluated in the 4×7 m tunnel.[5] The most promising configuration, shown in Fig. 2 as installed in the tunnel, was utilized for the present acoustic study. The structure consists of triangular vanes with a span of 0.6 m (23.6 in.) and a chord of 14.5 cm (5.7 in.), bent to form a 45 deg flap (see Fig. 3). These vanes were attached to the trailing edge of flat steel rails mounted 10 cm (4 in.) from the inside of the jet exit walls. The vanes were installed 0.6 m (23.6 in.) from each other on the rails, pointing alternately into and out of the flow. This vane configuration was aerodynamically effective in substantially reducing the pulsations. As shown in Fig. 1, only the flow pulsations existing at very low dynamic pressure remained. However, the vanes were a serious source of flow noise and, therefore, of concern for acoustic testing. The purpose of this Note is to present the results of an attempt to reduce the inherent noise of the vanes while retaining their pulsation reduction features.

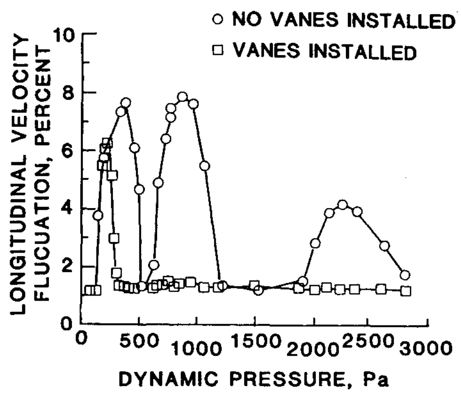

Fig. 1 Longitudinal velocity fluctuation in NASA Langley 4×7 m wind tunnel open-throat test section, with and without jet exit vanes installed (data from Ref. 5).

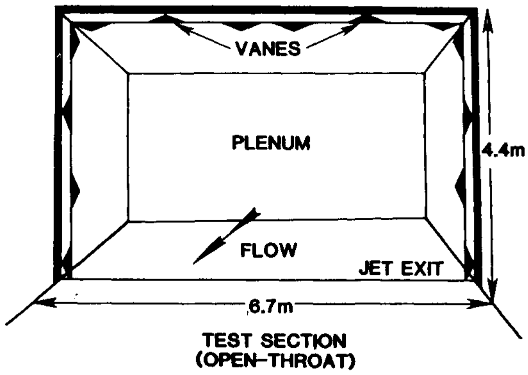

Fig. 2 Pulsation reduction vanes installed in jet exit.

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*Aerospace Engineer, Aeroacoustics Branch, Acoustics Division. Associate Member AIAA.

†Aerospace Engineer, Aeroacoustics Branch, Acoustics Division.

‡Aerospace Engineer, Rotorcraft Aerodynamics Office, Aerostructures Directorate, USAARTA (AVSCOM).